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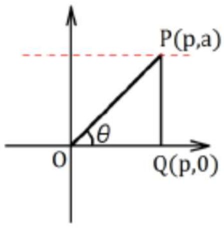
The ratio of the height to the base of a right triangle, and reducible set theoretical division by zero

In *x*-*y* coordinates, let the point of intersection between an arbitrary point P (p,a) in the first quadrant and a perpendicular line drawn from point P to the *x*-axis be represented as Q$(p,0)$. A right triangle ⊿OPQ is formed with ∠POQ = θ (refer to the following diagram).



Then, the ratio of the height to the base, tan θ, can be represented as

$$\tan\theta = \frac{\text{height}}{\text{base}} = \frac{a}{p}$$

Here, let point P be moved parallel to the *x*-axis (along the dotted red line in the diagram above), towards the *y*-axis. This implies P$(p \to +0, a)$. Then, the ratio of the height to the base, tan θ, becomes

$$\lim_{\theta \to \frac{\pi}{2}} \tan\theta = \lim_{p \to +0} \frac{a}{p} = +\infty$$

Let point P be moved further parallel to the *x*-axis until it is over the *y*-axis. Then, this implies P$(p = 0, a)$. Then, according to reducible set theoretical division by zero, the ratio of the height to the base, tan θ, becomes

$$\tan\frac{\pi}{2} = \frac{\text{height}}{\text{base}} = \frac{a}{p} = \frac{a}{0} = 0 \ldots a$$

This corresponds to the statement: "When the base is 0, the ratio of the height to the base is equal to 0, and the height *a* is left over as a remainder."

Of course, when $a = 1$,

$$\frac{\text{height}}{\text{base}} = \frac{a}{p} = \frac{1}{0} = 0 \ldots 1$$