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Division by zero and point mass in the principle of levers

Consider a rod of length L with no deformation that is thinner than any object of finite thickness and that has been horizontally placed. Let the point that internally divides the rod in the direction of its length be represented by P_0, and let the distance from point P_0 to point P_a on one end be represented by a, and let the distance from point P_0 to point P_b on the other end be represented by b. Let point P_o be the fulcrum, and let the force acting on point P_a (in the vertical downward direction) be represented by F_a , and let the force that acts on endpoint P_b as a result be represented by F_b (refer to the diagram below). Then, the lever ratio of a with respect to b in this system is

$$\frac{a}{b} \qquad (1)$$

The magnitude of force F_b (simply considered to be a scalar) is given by

$$F_b = \frac{a}{b} F_a \qquad (2)$$

Here, if we let $b \to 0$, then $F_b \to \infty$. According to the fundamental theorem of the division by zero,

$$b = 0 \Rightarrow F_b = 0 \qquad (3)$$

holds.

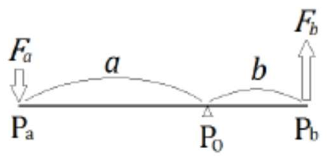

Figure: Class-1 lever system

This result agrees with experimental reality. In other words, this result states that when the location of the fulcrum P_0 is the same as the location of the endpoint P_b, then regardless of the magnitude of the force F_a applied at P_a, the force F_b that acts on endpoint P_b is zero. The fact that F_b never becomes infinite is something that everyone knows based on experience. Equation (3) implies that when $b = 0$, the force F_b and moment that act at endpoint P_b are zero. This implies that there is no moment generated at the point that lies at a radius of zero.
According to the definition, because $a = L – b$, Equation (2) can be expressed as

$$F_b = \frac{L - b}{b} F_a \qquad (4)$$

If we let $b = 0$, then

$$F_b = \frac{L-b}{b}F_a = \frac{L-0}{0}F_a = \frac{L}{0}F_a = 0 \times F_a \qquad (5)$$

Therefore, in the expression

$$F_b = \frac{L}{0}F_a \qquad (6)$$

in Equation (5), if we let $L \to 0$, this becomes a point mass. In addition, if Equation (4) is transformed into the following form

$$F_a = \frac{b}{L-b}F_b \qquad (7)$$

and $b = 0$, in other words

$$F_a = \frac{0}{L-0}F_b \qquad (8)$$

then if we let $L \to 0$, this becomes a point mass, and it is clear that $F_a = F_b = 0$. This can be interpreted as implying that the moment that acts on a point mass is zero. □