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Methods for applying division by zero and zero to the power of zero to an example of the second law of motion in Newtonian physics

Newton’s equation of motion can be expressed in simplified form as

$$ma = F \qquad (1)$$

assuming $m \neq 0$. Dividing both sides by a, we obtain

$$m\frac{a}{a} = \frac{F}{a} \qquad (2)$$

Note that if we are able to limit consideration to the case of $a \neq 0$, it is acceptable to express this as

$$m = \frac{F}{a} \qquad (3)$$

However, if we include the case $a = 0$, then this equation must be expressed in the form shown in Equation (2). This is because it is assumed that

$$\frac{a}{a} = 1 \qquad (4)$$

in the process of simplifying Equation (2) to Equation (3). However, Equation (4) only holds in the case $a \neq 0$. In the case $a = 0$, the value of the ratio a/a becomes

$$\frac{a}{a} = \frac{0}{0} = 0 \qquad (5)$$

Of course, the above equation is a result of division by zero. Based on the above result, we choose to use Equation (2) when both sides are divided by a. Then, when $a = 0$, the left-hand side of Equation (2) becomes

$$m\frac{a}{a} = m\frac{0}{0} = m \cdot 0 = m \cdot 0^{-1} = 0 \cdots m \qquad (6)$$

and the right-hand side of Equation (2) becomes

$$\frac{F}{a} = \frac{0}{0} = 0 \qquad (7)$$

This implies that we will not obtain the following illogical equation, which is what we would have obtained had we substituted $a = 0$ into Equation (3) by mistake:

$$m = \frac{F}{a} = \frac{0}{0} = 0 \qquad \therefore \quad m = 0 \qquad (8)$$

Furthermore, because we assume that $m \neq 0$ for the mass m, the remainder m appears as shown in Equation (6). This is due to reducible set theoretical division by zero, and this is not a statement that $m = 0$ as shown in Equation (8). In other words, this implies that it is not possible to determine the mass m in Newton's law of motion if it is not possible to determine the acceleration a that arises as a result of application of a certain force F. Of course, if the mass m is determined through a different method, then this value will agree with the remainder m in Equation (6).

Next, using the existence operation, Equation (1) can be rewritten as:

$$\left(\frac{m}{m}m\right) \cdot \left(\frac{a}{a}a\right) = \left(\frac{F}{F}F\right) \qquad (9)$$

Multiplying both sides of this equation by a^{-1} (using the existence operation, of course) gives:

$$\begin{aligned} \left(\frac{m}{m}m\right) \cdot \left(\frac{a}{a}a\right) \cdot \left(\frac{a}{a}a\right)^{-1} &= \left(\frac{F}{F}F\right) \cdot \left(\frac{a}{a}a\right)^{-1} \\ \left(\frac{m}{m}m\right) \cdot \left(\frac{a}{a}a\right)^{1-1} &= \left(\frac{F}{F}F\right) \cdot \left(\frac{a}{a}a\right)^{-1} \\ \left(\frac{m}{m}m\right) \cdot \left(\frac{a}{a}a\right)^{0} &= \left(\frac{F}{F}F\right) \cdot \left(\frac{a}{a}a\right)^{-1} \qquad (10) \end{aligned}$$

Next, we will show the case in which both sides of Equation (1) are multiplied by a^{-1} using the form in which the simplified general reciprocal, or in other words, the existence operation, has been omitted. This becomes:

$$\begin{aligned} m \cdot a \cdot a^{-1} &= F \cdot a^{-1} \\ m \cdot a^{1-1} &= F \cdot a^{-1} \\ ma^0 &= Fa^{-1} \qquad (11) \end{aligned}$$

If we set $a \neq 0$ in Equation (10) and Equation (11), then both equations become

$$m = \frac{F}{a}$$

in their simplified forms, which is the same form as Equation (3). However, when $a = 0$, the left-hand side in the simplified form of Equation (10) and Equation (11) becomes

$$ma^0 = m \cdot 0^0 = m \cdot 0 = m \cdot 0^{-1} = 0 \cdots m \qquad (12)$$

in both cases, and the right-hand side becomes

$$F \cdot a^{-1} = 0 \cdot 0^{-1} = 0^0 = 0 \qquad (13)$$

This is fundamentally the same solution as Equation (6) and Equation (7).

If we start with Equation (3) and multiply both sides by *a*, then we obtain

$$ma = \frac{F}{a}a \qquad (14)$$

If we further multiply both sides by a^{-1}, then we obtain

$$m \cdot a \cdot a^{-1} = \frac{F}{a} \cdot a \cdot a^{-1}$$

$$ma^0 = \frac{F}{a}a^0 \qquad (15)$$

As long as $a \neq 0$, Equation (15) is in the same form as Equation (3). However, when $a = 0$, Equation (3) is incorrect, and Equation (15) is in the correct form. In other words, we can say that Equation (3) applies in the narrow sense, and Equation (15) is its natural extension. This equation also implies that there are cases in which it is not possible to generate a finite acceleration *a* by applying a finite force *F* to a finite mass *m*. This may be similar to how it is not possible to move a fixed object by applying a force to it.