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Function remainder formula division by zero

Theorem: Let P(x) be the dividend function, Q(x) be the divisor function, A(x) be the quotient function, and R(x) be the remainder function. Then, if we let

$$P(x) = Q(x)A(x) + R(x)$$

in the equation

$$\frac{P(x)}{Q(x)} = \frac{Q(x)}{Q(x)}A(x) \cdots R(x)$$

for a value of a that satisfies Q(a) = 0, the following holds:

$$\frac{R(a)}{0} = 0 \cdots R(a)$$

□

Proof: According to the assumptions, because P(x) is the dividend function, Q(x) is the divisor function, A(x) is the quotient function, and R(x) is the remainder function, we have

$$P(x) = Q(x)A(x) + R(x) \qquad (1)$$

Therefore, this can be expressed as

$$\frac{P(x)}{Q(x)} = \frac{Q(x)}{Q(x)}A(x) \cdots R(x) \qquad (2)$$

Note that 0/0 = 0, and the expression is only reducible when Q/Q = 1 holds. Therefore, we choose not to reduce Equation (2).

Then, according to Equation (1),

$$Q(a) = 0 \Rightarrow R(a) = P(a) \qquad (3)$$

Therefore, if we let $x = a$ in Equation (2), the left-hand side becomes

$$\frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)} = \frac{P(a)}{0} = \frac{R(a)}{0} \qquad (4)$$

and the right-hand side becomes

$$\frac{Q(x)}{Q(x)}A(x) = \frac{Q(a)}{Q(a)}A(a) = \frac{0}{0}A(a) = 0 \cdot A(a) = 0 \cdots R(a) \qquad (5)$$

Therefore, according to Equation (4) and Equation (5), we obtain

$$\frac{R(a)}{0} = 0 \cdots R(a) \qquad (6)$$

□

Note that it is necessary to be aware that $A(a) = 0$ does not hold in general.

Consider function remainder formula division by zero in the following specific example. If we let the dividend function P(x) be

$$P(x) = 2x^2 - 3x + 1 \qquad (7)$$

and the divisor function Q(x) be

$$Q(x) = x + 1 \qquad (8)$$

then the quotient function A(x) becomes

$$A(x) = 2x - 5 \qquad (9)$$

and the remainder function R(x) becomes

$$R(x) = 6 \qquad (10)$$

Therefore, substituting Equation (7), (8), (9), and (10) into Equation (2), we obtain

$$\frac{2x^2 - 3x + 1}{x + 1} = \frac{x + 1}{x + 1}(2x - 5) \cdots 6 \qquad (11)$$

If we let $f(x)$ be equal to the left-hand side of Equation (11) and $g(x)$ be equal to the right-hand side, then when $x = b \neq -1$,

$$Q(b) \neq 0 \qquad (12)$$

Therefore, Equation (11) becomes

$$\frac{2b^2 - 3b + 1}{b + 1} = (2b - 5) \cdots 6 \qquad (13)$$

and

$$2b^2 - 3b + 1 = (b + 1)(2b - 5) + 6 \qquad (14)$$

holds, and satisfies the relationship in Equation (1).

When $x = -1$,

$$Q(-1) = 0 \qquad (15)$$

Therefore, $f(x)$ in Equation (11) becomes

$$f(-1) = \frac{2(-1)^2 - 3(-1) + 1}{(-1) + 1} = \frac{6}{0} \qquad (16)$$

and $g(x)$ becomes

$$g(-1) = \frac{(-1) + 1}{(-1) + 1}\{2 \cdot (-1) - 5\} = \frac{0}{0} \cdot (-7) = 0 \cdot (-7) = 0 \cdots 6 \qquad (17)$$

Therefore, based on Equation (16) and (17), we obtain

$$\frac{6}{0} = 0 \cdots 6 \qquad (18)$$

In other words, this implies that

$$6 = 0 \cdot 0 + 6 \qquad (19)$$

In this example,

$$A(-1) = -7 \qquad (20)$$

Therefore, $A(a) = 0$ does not necessarily hold.