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Remainder formula division by zero

Dividend a, divisor b, quotient c, and general remainder d can be represented as non-negative real numbers by

$$\frac{a}{b} = c \cdots d \qquad (1)$$

The general remainder d satisfies $0 \leqq d \leqq a$, and takes on a value that is equal to the minimum non-negative value when c is maximized. In particular, if $b \neq 0$, then $0 \leqq d < b$ is satisfied. Equation (1) can also be represented as

$$a = b \times c + d \qquad (2)$$

Here, subtracting d from both sides of Equation (2) and dividing both sides by b, we can transform this equation to obtain

$$\frac{a-d}{b} = \frac{b}{b} \cdot c \qquad (3)$$

Here, it is thought that $0/0 \neq 1$, and because only $b/b = 1$ is reducible, we choose not to reduce this expression.

Here, if we let $b = 0$, then according to Equation (2), $a = d$. Therefore, the left-hand side of Equation (3) becomes

$$\frac{a-d}{b} = \frac{a-a}{0} = \frac{0}{0} \qquad (4)$$

and the right-hand side becomes

$$\frac{b}{b} \cdot c = \frac{0}{0} \cdot c \qquad (5)$$

Therefore, based on Equation (4) and (5), we obtain

$$\frac{0}{0} = \frac{0}{0} \cdot c \qquad (6)$$

Therefore, we transform this equation into

$$(c-1)\frac{0}{0}=0 \qquad (7)$$

If we assume that $c \neq 1$, we can divide both sides by $c - 1$ and transform this equation into

$$\frac{0}{0}=\frac{0}{c-1} \qquad (8)$$

and obtain

$$\frac{0}{0}=0 \qquad (9)$$

If we let $a = d = 0$ in Equation (1), then a comparison with Equation (9) shows that $c = 0$, and that this does not contradict the assumption that $c \neq 1$. Conversely, if we let $a = d \neq 0$, then because d takes on the maximum value out of all possible values for d, this implies that the possible value for c consists of the minimum non-negative value. Based on this, we obtain $c = 0$. Of course, this does not contradict the assumption that $c \neq 1$ either. □