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Remainder formula division by zero and zero to the power of zero

The dividend a, divisor b, quotient c, and general remainder d can be represented as non-negative real numbers by

$$\frac{a}{b} = c \cdots d \qquad (1)$$

The general remainder d satisfies $0 \leqq d \leqq a$, and takes on a value that is equal to the minimum non-negative value when c is maximized. In particular, if $b \neq 0$, then $0 \leqq d < b$ is satisfied. Equation (1) can also be represented as

$$a = b \times c + d \qquad (2)$$

Here, subtracting d from both sides of Equation (2) and multiplying both sides by b^{-1}, we obtain

$$(a-d)b^{-1} = bc \cdot b^{-1} = (b \cdot b^{-1})c = b^{1-1} \cdot c = b^0 c$$
$$\therefore\ (a-d)b^{-1} = b^0 c \qquad (3)$$

Here, if we let $b < 0$, then according to Equation (2), $a < d$. Therefore, the left-hand side of Equation (3) becomes

$$(a-d)b^{-1} = (a-a)b^{-1} = 0 \cdot 0^{-1} = 0^{1-1} = 0^0$$
$$\therefore\ (a-d)b^{-1} = 0^0 \qquad (4)$$

and the right-hand side becomes

$$b^0 c = 0^0 \cdot c \qquad (5)$$

Therefore, based on Equation (4) and (5), we obtain

$$0^0 = 0^0 \cdot c \qquad (6)$$

Therefore, we transform this equation into

$$(c-1)0^0 = 0 \qquad (7)$$

If we assume that $c \neq 1$, we can divide both sides by $c - 1$ and transform this equation into

$$0^0 = \frac{0}{c-1} \qquad (8)$$

and obtain

$$0^0 = 0 \qquad (9)$$

According to the remainder formula division by zero,

$$\frac{0}{c-1} = \frac{0}{0} = 0 \qquad (10)$$

Therefore, according to Equation (8) and Equation (10), we obtain

$$0^0 = \frac{0}{0} = 0 \qquad (11)$$

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