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Hiroshi Michiwaki

Tangent and Division by Zero

Theorem Given a real number a,

$$\tan\frac{\pi}{2} = \frac{a}{0} = 0$$

Proof On an xy-coordinate plane, concentric circles C_1 and C_2 of radii r_1 and r_2, respectively, are drawn with their center at the origin O (see Fig. 1). A point $P_1(a_1,b_1)$ is selected on the circumference of C_1 and, starting from the origin, a line segment OP is drawn that makes radial angle θ with the x-axis. Then, that segment is extended to C_2, and the intersection point is called $P_2(a_2,b_2)$. Moreover, it is assumed that $r_1 \leqq r_2$, as in Fig. 1. Therefore, $\Delta r = r_2 - r_1 \geqq 0$.

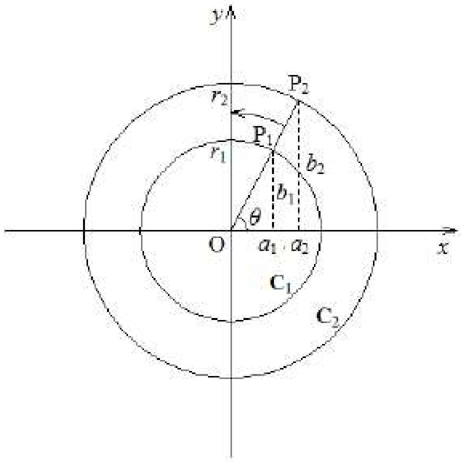

Fig.1

Thus,

$$\tan\theta = \frac{b_1}{a_1} \qquad (1)$$

$$\tan\theta = \frac{b_2}{a_2} \qquad (2)$$

Then, if $\theta = \pi/2$, it is clear that $b_1 = r_1 \wedge b_2 = r_2$; thus, Equations (1) and (2) can be expressed as

$$\tan\theta = \frac{b_1}{a_1} \Rightarrow \tan\frac{\pi}{2} = \frac{r_1}{0} \qquad (3)$$

$$\tan\theta = \frac{b_2}{a_2} \Rightarrow \tan\frac{\pi}{2} = \frac{r_2}{0} \qquad (4)$$

Therefore, it is clear from (3) and (4) that

$$\tan\frac{\pi}{2} = \frac{r_1}{0} = \frac{r_2}{0} \qquad (5)$$

which yields

$$\frac{r_2}{0} - \frac{r_1}{0} = \frac{1 \cdot r_2}{0} - \frac{1 \cdot r_1}{0} = \frac{1}{0}r_2 - \frac{1}{0}r_1 = \frac{1}{0}(r_2 - r_1) = \frac{1}{0}\Delta r = \frac{\Delta r}{0} = 0 \qquad (6)$$

Then, as long as $\Delta r \geqq 0$, multiplying both sides of (6) by –1 leaves (6) invariant for any Δr. Thus, for the real number a,

$$\frac{a}{0} = 0 \qquad (7)$$

Consequently,

$$\tan\frac{\pi}{2} = \frac{a}{0} = 0$$

QED.