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Feasibility of Division by Zero Based on Fixed-Point Theorems

Theorem Given a zero vector **0**, the following is true:

$$\frac{\mathbf{0}}{0} = \mathbf{0}$$

and so is 0/0 = 0.

Proof Using the unit position vector $\boldsymbol{e}_r$, the norm $|\boldsymbol{r}|$ of the position vector $\boldsymbol{r}$, and the tangential velocity vector $\boldsymbol{v}$, the acceleration vector $\boldsymbol{\omega}$ is expressed as

$$\boldsymbol{\omega} = \boldsymbol{e}_r \times \frac{\boldsymbol{v}}{|\boldsymbol{r}|} \qquad (1)$$

A closed disk $B^2 = \{(x, y)|x^2 + y^2 \leq 1\}$ is now considered on a flat surface R^2, and it is assumed that it is given in polar coordinates $\{r, \theta\}$.

Then, according to L.E.J. Brouwer's fixed-point theorem, continuous map f from the closed disk B^2 onto itself has at least one fixed point P. According to Poincare−Hopf fixed-point theorem, the tangential vector field contains at least one zero point **0**; therefore, there is a point on the disk B^2 where the tangential velocity vector $\boldsymbol{v} = \mathbf{0}$, which can be established as the origin (the center of the disk B^2), and in this case, the angular velocity vector $\boldsymbol{\omega}$ can be said to be $\boldsymbol{\omega} = \mathbf{0}$. Consequently, (1) becomes

$$\boldsymbol{\omega} = \boldsymbol{e}_r \times \frac{\boldsymbol{v}}{|\boldsymbol{r}|} = \boldsymbol{e}_r \times \frac{\mathbf{0}}{|\boldsymbol{r}|} = \frac{\boldsymbol{e}_r \times \mathbf{0}}{r} = \frac{\mathbf{0}}{r} = \mathbf{0} \qquad (2)$$

In addition, as the fixed point P is the origin, the position vector $\boldsymbol{r}$ in (2) is $\boldsymbol{r} = 0$, and thus

$$\boldsymbol{\omega} = \boldsymbol{e}_r \times \frac{\boldsymbol{v}}{|\boldsymbol{r}|} = \boldsymbol{e}_r \times \frac{\mathbf{0}}{|\mathbf{0}|} = \frac{\boldsymbol{e}_r \times \mathbf{0}}{0} = \frac{\mathbf{0}}{0} = \mathbf{0} \qquad (3)$$

Of course, if only the size of the zero vector **0** is considered, it can be treated as the scalar quantity 0 without any issues, yielding

$$\frac{0}{0} = 0 \qquad (4)$$

QED

Thus, fixed-point theorems arguably ensure that division by zero can be performed, and the zero vector division theorem described above easily leads to the fundamental theorem of division by zero, 0/0=0. Conversely, the above theorem is the vector extension of the fundamental theorem of division by zero, 0/0=0.