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Closed-Point Dimension and Division by Zero

"Closed points" are Euclidean points. That is, other than the indicating position, they do not possess any mathematical characteristics such as size, direction, or slope. Moreover, in Euclidean dimensions, lines are considered to be 1-dimensional; surfaces, 2-dimensional; and volumes, 3-dimensional. The dimension is thus defined by the number of 1-dimensional independent variables. What is the closed-point dimension? Closed points do not possess any mathematical characteristics. Do they have a dimension? If they do, are they 0-dimensional? Hypothetically, if so, *why* are closed points are 0-dimensional? The Euclidean dimension D_E is expressed by the following equation:

$$D_E = \frac{\log N(\gamma)}{\log \gamma} \qquad (1)$$

where $1/\gamma$ is the scale in each spatial direction of unit length and $N(\gamma)$ is the number of fractals, expressed as $N(\gamma) = \gamma^{DE}$. Closed points have no size and cannot be made smaller; thus, the scale remains 1 as it originally was. Hence, as $1/\gamma = 1$, $\gamma = 1$. Furthermore, as there is only one fractal, which is the original point, the number of fractals is of course N(1) = 1. Therefore, (1) becomes

$$D_E = \frac{\log N(\gamma)}{\log \gamma} = \frac{\log N(1)}{\log 1} = \frac{\log 1}{0} = \frac{0}{0} = 0 \qquad (2)$$

resulting in

$$D_E = 0 \qquad (3)$$

for the closed-point dimension. Equation (2) makes use of the fundamental theorem of division by zero, 0/0 = 0. Furthermore, it should be noted that even if it was hypothetically possible to let γ be large ($\gamma > 1$) and reduce the scale $1/\gamma$, the number of fractals $N(\gamma)$ would remain 1, as the size of a closed point is zero, and the result of (3) would not change. In addition, if the number of fractals $N(\gamma) = \emptyset = 0$, due to the fact that closed points do not possess any mathematical characteristics other than position, then

$$\log N(\gamma) = \log 0 = 0 \qquad (4)$$

and, as expected, (3) holds true. However, log 0 = 0 in (4) depends on the theorem in the Division by Zero paper #29, "log 0 = 0 and Division By Zero."