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NEW MEANINGS OF THE DIVISION BY ZERO AND INTERPRETATIONS ON $100/0 = 0$ AND ON $0/0 = 0$

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Abstract: In this paper, we shall give simple and natural, however, the surprising identities $100/0 = 0$ and $0/0 = 0$ by a natural extension of fractions with the concept of Tikhonov regularization using the theory of reproducing kernels. We shall give interpretations of the results.

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1. Introduction

In order to state the essential idea in elementary way, we shall consider the fractions on the real numbers. Then, by a natural extension of the fraction

$$\frac{b}{a} \tag{1.1}$$

for any numbers a and b, we shall, in particular, show the surprising results

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$$\frac{100}{0} = 0, \frac{0}{0} = 0. \tag{1.2}$$

However, in our recent paper [3] or the concept of the Moore-Penrose generalized inverse, the above results are clear - indeed, the results (1.2) themselves were found in a very general vector version in Saitoh ([7]), however, many mathematicians do not admit the results. So, we would like to show the results in the elementary means and we shall give several interpretations of the results. Then, the results may be considered as very natural ones. Furthermore, the results will create many models of problems in analysis and physics on the universe.

2. Definition of General Fractions

We shall give the definition first: for any real numbers a and b, the general fractions

$$\frac{b}{a} \tag{2.1}$$

are defined by the following way:

For any fixed $\lambda > 0$, the minimum member of the Tikhonov function in x

$$\lambda x^2 + (ax - b)^2; \tag{2.2}$$

that is,

$$x_\lambda(a, b) = \frac{ab}{\lambda + a^2} \tag{2.3}$$

is the fraction in the sense of Tikhonov. By taking the limit

$$\lim_{\lambda \to +0} x_\lambda(a, b) = \frac{b}{a}, \tag{2.4}$$

we shall define the general fractions (2.1).

Note that, for $a \neq 0$, the definition (2.1) is the same as the ordinary sense, however, when $a = 0$, we obtain the desired results, since $x_\lambda(0, b) = 0$, always.

For the general theory of the Tikhonov regularization and many applications, see the cited references.

Mathematicians will expect if there exists some realization of our mathematics in the real world models.

So, we wonder: does there exist some real examples supporting the above results ([8])?

Recently, Takahashi ([9]) established a simple and natural interpretation of (1.2) by analyzing any extensions of fractions and by showing the complete characterization for such property (1.2). Furthermore, he examined several fundamental properties of the general fractions. His result will show that our mathematics says that the results (1.2) should be accepted as natural ones. However, the results will be curious for our feelings and so, the above question will be vital still as a very important problem.

Dr. M. Dalla Riva proved simply and directly the following:

Theorem. *Let F be a function from $\mathbf{R} \times \mathbf{R}$ to $\mathbf{R}$ such that*

$$F(a,b)F(c,d) = F(ac, bd)$$

for all

$$a, b, c, d \in \mathbf{R}$$

and

$$F(a,b) = \frac{a}{b}, \quad a, b \in \mathbf{R}, b \neq 0.$$

Then, we obtain, for any $a \in \mathbf{R}$

$$F(a,0) = 0.$$

Proof. We have

$$\begin{aligned} F(a,0) = F(a,0)1 = F(a,0)\frac{2}{2} &= F(a,0)F(2,2) = F(a \cdot 2, 0 \cdot 2) \\ &= F(2a,0) = F(2,1)F(a,0) = 2F(a,0). \end{aligned}$$

Thus $F(a,0) = 2F(a,0)$ which implies the desired result $F(a,0) = 0$ for all $a \in \mathbf{R}$. $\square$

Several mathematicians pointed out to the authors that the notations of $100/0$ and $0/0$ are not good for the sake of the generalized sense, however, there does not exist other natural and good meaning for them. Why should we need and use any new notations for involving the notations? We should use the usual notations, we think so.

3. An Interpretation of $\frac{1}{0} = 0$

We will consider that the result $\frac{1}{0} = 0$ is a curious one, because

$$\lim_{x \to +0} \frac{1}{x} = +\infty$$

and meanwhile,

$$\lim_{x \to -0} \frac{1}{x} = -\infty.$$

However, recall the graph of the function

$$y = \frac{1}{x}.$$

Then, we see the beautiful point 0 as the center of the graph or 0 is the mean value of $+\infty$ and $-\infty$, and the mean value is the basic property in mathematics and in the universe.

However, when we image the graph as a big bang at the origin, then we will be able to enjoy the graph and then we can expect the half some world from the negative real axis.

4. A Physical Interpretation of $\frac{0}{0} = 0$

We shall give a simple physical model showing the result $\frac{0}{0} = 0$. We shall consider a disc with $x^2 + y^2 \leq a^2$ rolling uniformly with a positive constant angle velocity ω with the center at the origin. Then we see, at the only origin, $\omega = 0$ and at all other points, ω is a constant. That is, the velocity and the radius r are zero at the origin. This will mean that, in the general formula

$$v = r\omega,$$

or, in

$$\omega = \frac{v}{r}$$

we have at the origin,

$$\frac{0}{0} = 0.$$

We shall not be able to obtain the result from

$$\lim_{r \to +0} \omega = \lim_{r \to +0} \frac{v}{r},$$

because it is the constant.

5. By the Newton's Law

We shall recall the fundamental law by Newton:

$$F = c\frac{m_1 m_2}{r^2} \tag{5.1}$$

for two masses m_1, m_2 with a distance r and a constant c. Of course,

$$\lim_{r \to +0} F = \infty, \tag{5.2}$$

however, as in our fraction

$$F = 0 = c\frac{m_1 m_2}{0}. \tag{5.3}$$

Of course, here, we can consider the above interpretation for the mathematical formula (5.1) as the new interpretation (5.3). In the ideal case, when the two masses are at point, the force F will not be positive, it will be reduced to zero.

For the Coulomb's law, see similar formulas.

Furthermore, as well-known, the bright at a point at the distance r from the origin is given by the formula

$$B = k\frac{P}{r^2}, \tag{5.4}$$

where k is a constant and P is the amount of the light. Of course, we have, at the infinity:

$$B = 0. \tag{5.5}$$

Then, meanwhile, may we consider as

$$B = 0 \tag{5.6}$$

at the origin $r = 0$? Then we can obtain our formula

$$k\frac{P}{0} = 0,$$

as in our new formula.

We found several very interesting results from several physical formulas that our general fractions are very reasonable physically. We see that such general interpretations exist in our beings around.

6. Analytic Function Theory, Singularity

We shall recall the elementary function

$$W(z) = \exp \frac{1}{z} \tag{6.1}$$

$$= 1 + \frac{1}{1!z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + \cdots.$$

This function has an essential singularity around the origin. However, surprisingly enough, we have

$$W(0) = 1. \tag{6.2}$$

From this fact, we will have a great world for singularity points problems of analytic functions, widely and deeply. Recall Picard's great theorem for the serious meanings.

7. An Interpretation of $0 \times 0 = 100$ from $100/0 = 0$

The expression $100/0 = 0$ will represent some division by the zero in a sense, not the usual one, and so, we will be able to consider some product in the sense $0 \times 0 = 100$.

We shall show such interpretation.

We shall consider same two masses m, however, their constant velocities v for the origin are the same in the real line, in the symmetry way: We consider the moving energy product E^2,

$$\frac{1}{2}mv^2 \times \frac{1}{2}m(-v)^2 = E^2. \tag{7.1}$$

We shall consider at the origin and we assume the two masses stop at the origin (possible in some case). Then, we can consider, formally

$$0 \times 0 = E^2. \tag{7.2}$$

The moving energies turn to other energies, however, we can obtain some interpretation as in the above.

8. Capillary Pressure in a Narrow Capillary Tube

In a narrow capillary tube saturated with fluid such as water, the capillary pressure is simply expressed as follows:

$$Pc = \frac{2\sigma}{r}, \tag{8.1}$$

where Pc is capillary pressure (suction pressure), σ is surface tension, and r is radius. If r is zero, there is no pressure. However Pc shows infinity, in the common meaning.

This simple equation is based on the Laplace-Young equation

$$P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \tag{8.2}$$

where R_1 and R_2 are two principal radii of curvature at any point on the surface of a droplet or a bubble and in the case of spherical form, $R_1 = R_2 = R$. For a spherical bubble the pressure difference across the bubble film is zero as the pressure is the same on both sides of the film. The Laplace-Young equation reduces to

$$\frac{1}{R_1} + \frac{1}{R_2} = 0. \tag{8.3}$$

On the other hand, when the diameter of a bubble is decreased and becomes 0 ($R = 0$), the bubbles collapse, and enormous energy is generated. The accumulated free energy in the bubble is released instantaneously.

M. Yamane referred to the Doppler effects that will have many related problems and should examine them physically. We hope our new formula may contribute to many models of physical problems.

Meanwhile, Professor Igarashi ([4]) showed the natural interpretation for our general fractions with computer languages by natural algorithms. In particular, he showed that the general fractions are natural ones from the viewpoint of computer algorithms.

For the above problems, the statements are not so simple, and so we expect their publication by separate covers.

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